Physics ⇌ Problem solving

e.g., cutting edge research, reading a book, etc.

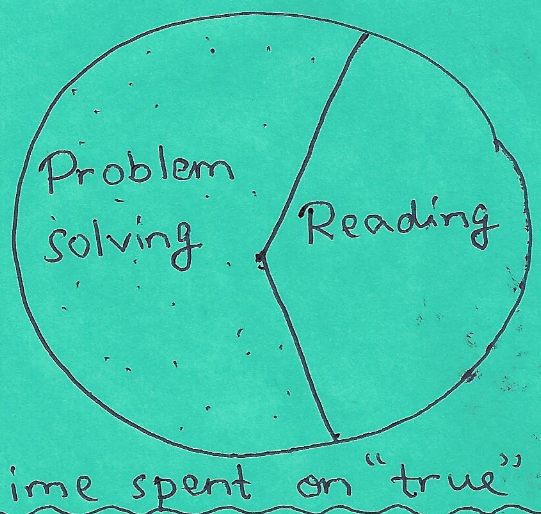
True test of understanding something

⇕

Ability to solve problems on it.

{ communicate & collaborate

Reading is "necessary but not sufficient" condition for learning.



Time spent on "true" learning

Strategies for "Problem Solving"

- Diagrams
  - Clearly label all the relevant quantities (viz., forces, masses, flux, etc)

"has the potential to change hopelessly complicated problem into a 'near-trivial' one".

- Unknown & the known quantities
  - Clearly write down all these quantities.

e.g., 4 unknowns & only 2 physical facts $\Rightarrow$ 2 missing facts.

- Solve symbolically
  - → Quicker.
  - → Less prone to mistakes / Easy to detect mistakes.
  - → solve problem once & for all.
  - → physically appealing (general dependence of answer on various given quantities is evident)
  - → Allows for checking units → programmable
  - → special cases can be easily verified.
- Consider Units & dimensional validity
- Check order of magnitude of numerical answer

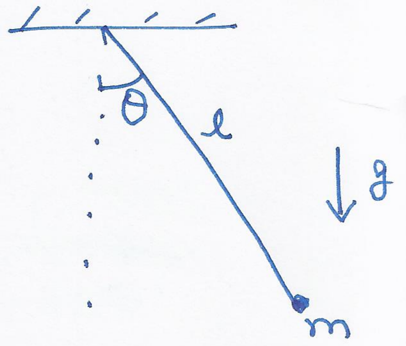
"Fermi Problem" $\sim 10^x$

- Plot.

Dimensional Analysis

Ex-1 Mass m hangs from a massless string of length l under earth's gravity. Can we say something about frequency of oscillations?

Known	unknown
g, l, m, θ	ω



$$[g] = L^1T^{-2}, \quad [m] = M^1, \quad [l] = L^1$$

$$[\theta] = M^0L^0T^0.$$

Let $\omega = m^a g^b l^c \cdot \underbrace{f(\theta_0)}_{\text{dimensionless factor}}$

$$\Rightarrow T^{-1} = M^a (L^1T^{-2})^b L^c \qquad \begin{cases} -2b = -1 \\ b + c = 0. \end{cases}$$

$$\Rightarrow a = 0, \; b = +\frac{1}{2}, \; c = -\frac{1}{2}$$

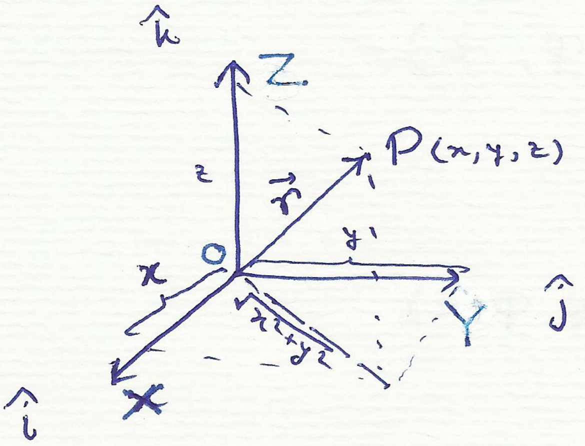
$$\therefore \quad \omega = f(\theta_0)\sqrt{\frac{g}{l}}.$$

(Independent of mass !).

Not doable using dimensional analysis.
$\left\{ \begin{array}{l} \cdot\ f(\theta_0) \simeq 1 \text{ for small oscillations.} \\ \cdot\ f(\theta_0) = 1 - \frac{\theta_0^2}{16} + \ldots \text{ for large } \theta_0. \end{array} \right.$

Coordinate systems

Cartesian



Cartesian system (3D)

$-\infty \leq x, y, z \leq +\infty$

Recap.

Angles of $\vec{r}$ with the axes

x-axis : α $\qquad x = r\cos\alpha$

y-axis : β $\qquad y = r\cos\beta$

z-axis : γ $\qquad z = r\cos\gamma$

$$x^2 + y^2 + z^2 = r^2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = r^2.$$

$$\Rightarrow \boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.}$$

Direction cosines:

$$l = \frac{x}{|\vec{r}|}, \quad m = \frac{y}{|\vec{r}|}, \quad n = \frac{z}{|\vec{r}|}.$$

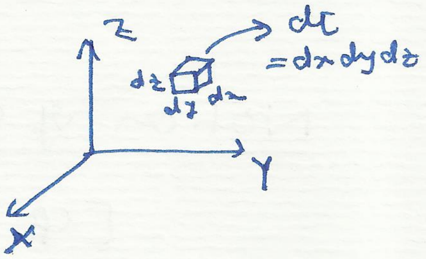
$$\left(= \frac{\vec{r}\cdot\hat{i}}{|\vec{r}||\hat{i}|}\right) \quad \left(= \frac{\vec{r}\cdot\hat{j}}{|\vec{r}||\hat{j}|}\right) \quad \left(= \frac{\vec{r}\cdot\hat{k}}{|\vec{r}||\hat{k}|}\right)$$

Direction ratio's:

x, y, z

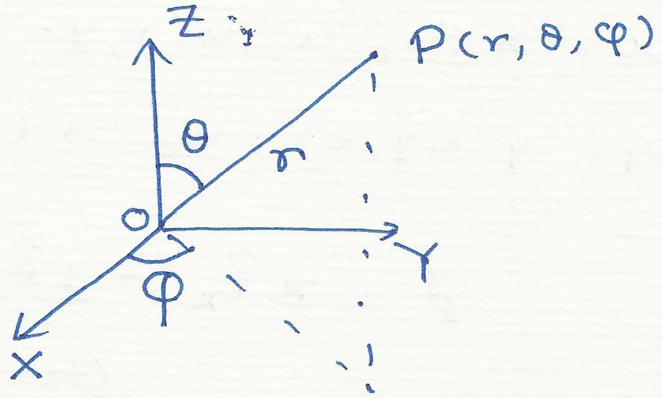
Surface elements : $dx\,dy$, $dy\,dz$, $dz\,dx$

volume element : $dx\,dy\,dz$.



Spherical polar coordinate

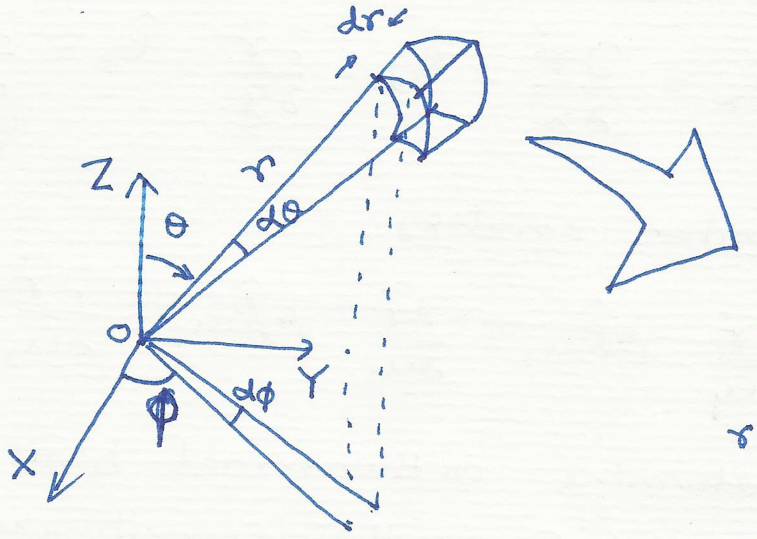
$$(r, \theta, \phi)$$

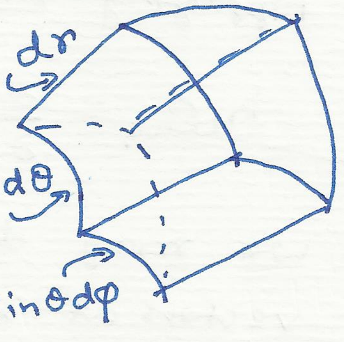


$0 \leq r \leq \infty$

$0 \leq \theta \leq \pi$

$0 \leq \phi \leq 2\pi$





$\therefore$ Volume element $(d\tau)$ $= r^2 \sin\theta \, d\theta \, d\phi \, dr$.

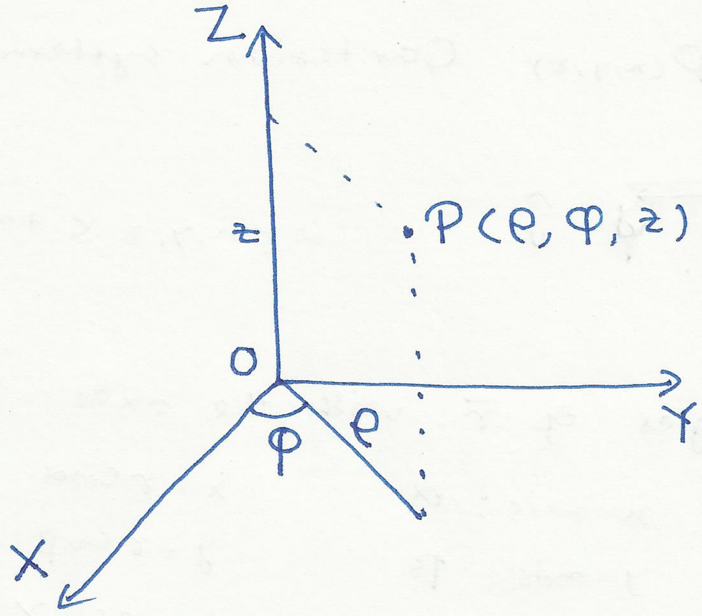
Check dimensions : $[d\tau] = L^3$.

Scale factors

$$\boxed{\begin{aligned} h_r &= 1 \\ h_\theta &= r \\ h_\phi &= r\sin\theta \end{aligned}}$$

Cylindrical polar coordinates

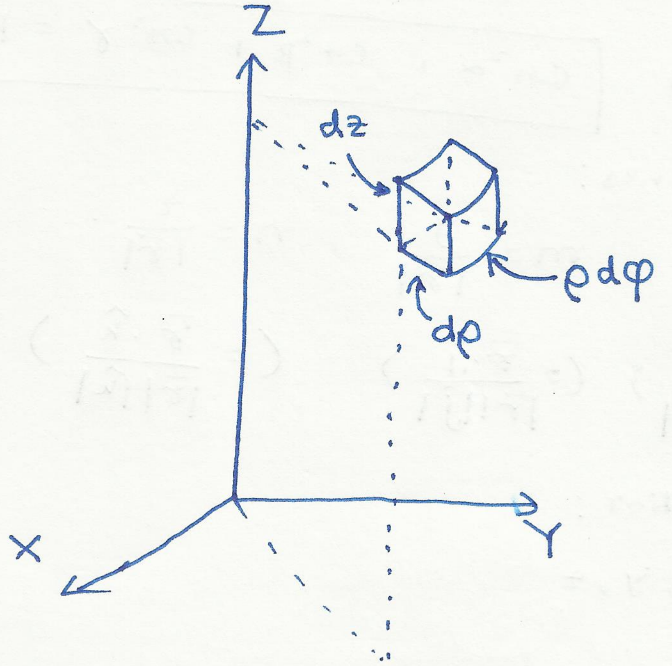
(ρ, φ, z)



$0 \le \rho \le \infty$

$0 \le \varphi \le 2\pi$

$-\infty \le z \le \infty$



Scale factors

$h_\rho = 1$

$h_\varphi = \rho$

$h_z = 1$

volume element $(d\tau) = \rho\, d\rho\, d\varphi\, dz$.

Notion of curvilinear coordinates (u_1, u_2, u_3)

$[\varphi] \neq L$.

But φ is a coordinate.

volume element

$d\tau = h_1 h_2 h_3\, du_1\, du_2\, du_3$

Curvilinear coordinates

May be derived from a set of Cartesian coordinates by using "locally invertible" (a one-to-one map) transformation.

$$x = x(u_1, u_2, u_3),\ y = y(u_1, u_2, u_3),\ z = z(u_1, u_2, u_3)$$

$$u_1 = u_1(x, y, z),\ u_2 = u_2(x, y, z),\ u_3 = u_3(x, y, z)$$

$$\vec{r} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$$

(identify $\hat{e}_x, \hat{e}_y, \hat{e}_z \equiv \hat{i}, \hat{j}, \hat{k}$).

where, $\hat{e}_x = \dfrac{\partial \vec{r}}{\partial x}$, $\hat{e}_y = \dfrac{\partial \vec{r}}{\partial y}$, $\hat{e}_z = \dfrac{\partial \vec{r}}{\partial z}$.

Applying similar analogy,

$$\cancel{h_1\hat{e}_{u_1} = \frac{\partial \vec{r}}{\partial u_1},\ h_2\hat{e}_{u_2} = \frac{\partial \vec{r}}{\partial u_2},\ h_3\hat{e}_{u_3} = \frac{\partial \vec{r}}{\partial u_3}}.$$

we may determine orthonormal basis vectors at a point P for the curvilinear system.

In general the basis vectors are

$$\vec{h}_1 = \frac{\partial \vec{r}}{\partial u_1},\ \vec{h}_2 = \frac{\partial \vec{r}}{\partial u_2},\ \vec{h}_3 = \frac{\partial \vec{r}}{\partial u_3}.$$

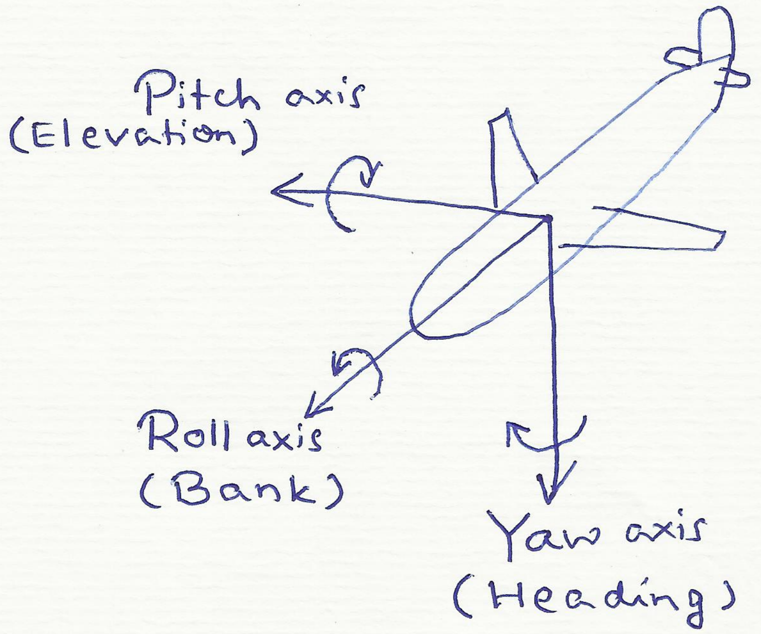
- [ ] $\neq$ L always
- may not be orthogonal at all points.

<u>Lamé coefficients</u>: $h_1 = |\vec{h}_1|$, $h_2 = |\vec{h}_2|$, $h_3 = |\vec{h}_3|$.

$\therefore$ Orthonormal curvilinear basis vectors are, $\hat{e}_{u_1} = \dfrac{\vec{h}_1}{h_1}$, $\hat{e}_{u_2} = \dfrac{\vec{h}_2}{h_2}$, $\hat{e}_{u_3} = \dfrac{\vec{h}_3}{h_3}$.

Note: $ds^2 = h_1^2 du_1^2 + h_2^2 du_2^2 + h_3^2 du_3^2$.

Additional ways to represent



$$(\theta_E, \theta_B, \theta_H)$$

$$(\theta_P, \theta_R, \theta_Y)$$

Tait-Bryan convention