Preliminaries – I
(Quantum Mechanics)

- Bohr versus Einstein

- "You can know everything about the system but not its parts"

- "Entanglement"

- Both classical mechanics & quantum mechanics have mathematically abstract notions.

- Developed intuition about classical mechanics due to everyday needs for survival. Quantum mechanics on the other hand deals with objects which we cannot probe using our five senses.

    However, correspondence principle holds and in suitable limits,

Quantum mechanics
↓
Classical mechanics

Differences

(A.) Different abstractions

→ State in quantum mechanics
Mathematical object with a logical structure different from classical mechanics.

(B.) State and measurement

Classical mechanics : labels describing state of a system are the same as the ones in a measurement.

Quantum mechanics : They are two different things and the relationship between them is subtle & non-intuitive.

Summary of Discussion on initial developments in Quantum Mechanics

Photoelectric effect

↓

Idea of light "quanta"
(photons)

↓

Planck's hypothesis

↓

Bose statistics → Explanation of ob ved black body spectrum

Davisson-Germer experiment
(particles have wave properties)

Photoelectric effect
(light has particle like properties)

⇓

wave-particle duality, $\lambda = \frac{h}{p}$
(de Broglie)

Quantum mechanical systems/objects have both wavelike as well as particle like properties.

→ But neither wave nor particle.

→ Of their own kind.

$\Psi(x,t)$: wavefunction; containing all information regarding the system.

(or, $|\Psi\rangle$)

$$\hat{H}\,\Psi(x,t) = i\hbar \frac{\partial}{\partial t}\Psi(x,t).$$

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right]\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t)$$

Schrödinger time dep. equation

Background

Lagrangian $\quad L(q_i, \dot{q}_i; t) = T - V$

$$\downarrow$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

$$\downarrow$$

Hamiltonian $\quad H(q_i, p_i) = \sum_i \dot{q}_i p_i - L$

$$\downarrow$$

$$\dot{p}_i = -\frac{\partial H}{\partial q_i}$$

$$\dot{q}_i = \frac{\partial H}{\partial p_i}$$

q_i & p_i are canonically conjugate variables

In Quantum mechanics, q_i & p_i are operators, s.t.,

$$[q_i, p_i] = i\hbar$$

$$\Downarrow$$

$$\Delta q_i \, \Delta p_i \geq \frac{\hbar}{2}$$

Δq_i is root mean squared deviation in q_i, i.e.,
$\Delta q_i = \sqrt{\langle (q_i - \langle q_i \rangle)^2 \rangle}$

Method of seperation of variables

$$\Psi(x,t) = \psi(x)\,T(t).$$

$$\Rightarrow -\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

$$\downarrow$$

$$\frac{1}{\psi(x)T(t)}\left[-\frac{\hbar^2}{2m}T(t)\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x)T(t)\right]$$

$$= \frac{1}{\psi(x)T(t)}\, i\hbar\,\psi(x)\frac{dT(t)}{dt}.$$

$$\Rightarrow -\frac{\hbar^2}{2m}\frac{1}{\psi(x)}\frac{d^2\psi(x)}{dx^2} + V(x) = \frac{i\hbar}{T(t)}\frac{dT(t)}{dt}$$

L.H.S. is a function of x only

while R.H.S. is a function of t only

$\therefore$ for above equation to be valid, L.H.S. = R.H.S. = Constant

Identify the constant as E.

$$\therefore\quad -\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x).$$

$$\& \quad \frac{i\hbar}{T(t)}\frac{dT(t)}{dt} = E \iff T(t) = Ce^{-\frac{iEt}{\hbar}}.$$

Time independent Schrödinger equation.