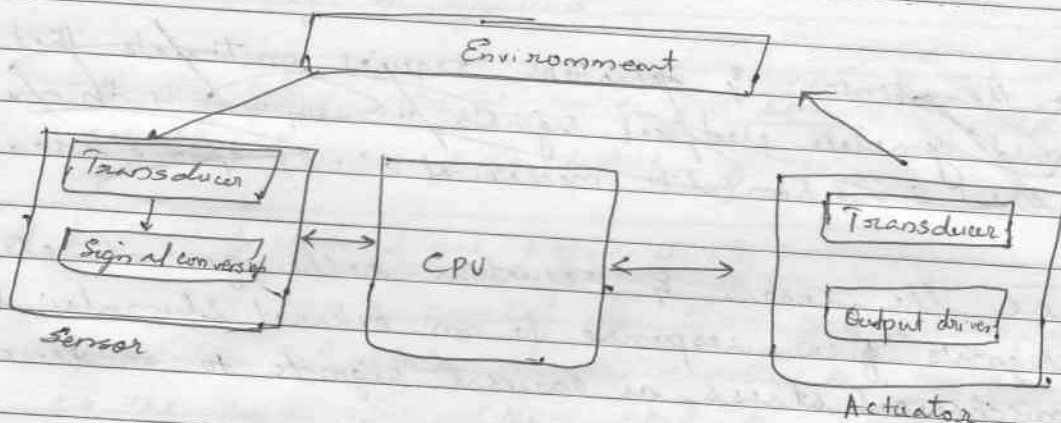


Sensors & Actuators:

Sensor: A device which receives and responds to an external signal or stimulus.

Actuator: A device responsible for mechanical motion or action.

Example: Accelerometers

- : devices that measure acceleration
- : also measures position and displacement
- : measured in units of m/s^2 or G-force $\rightarrow 1g$ force corresponds to $9.8 m/s^2$.
- : can sense either static or dynamic forces of acceleration
- : Static force - gravity
- : dynamic: vibrations and movement.

Accelerometers: Six Sensing Functions

1. Movement
2. Vibrations - High sensitivity and high frequency accelerometry
3. Fall - identify large impacts - Integrated into HDD's ^{systemic activity}
4. Positioning - Complex algorithm \rightarrow navigation in car, gps etc
5. Tilt - e-compass, text scrolling, image processing
6. Shock - Shipping & handling monitoring, black-box event recorders etc

i.e. the sensors & actuators require materials that could generate electrical signals in response to an external condition (like mechanical stress) & vice-versa.

i.e. the sensors & actuators either generate electrical signals in response to an external stimulus like mechanical stress or convert signals to mechanical movements.

Here piezoelectric materials play an important piezoelectric role, since these materials have linear electromechanical interaction between mechanical and the electrical responses (stimulus).

Basis of Piezoelectric effect:

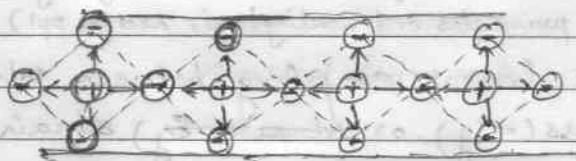
Direct piezoelectric effect: Forces applied to certain crystals generate a charge on the surface (and hence a voltage) which is proportional to the applied stress force.

Inverse piezoelectric effect: the material is deformed when electric voltage is applied.

i.e. the effect can be used for sensing (direct piezo) and actuation (inverse piezoelectric effect).

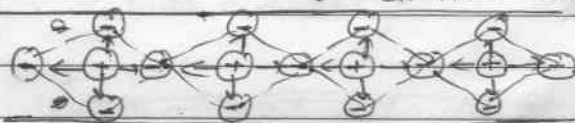
What kind of crystals show piezoelectricity?

→ For a crystal to show piezoelectric effect, the application of stress should produce a non-zero polarization i.e. the symmetry of the crystal is critical.



$$\sum \vec{P}_i = 0$$

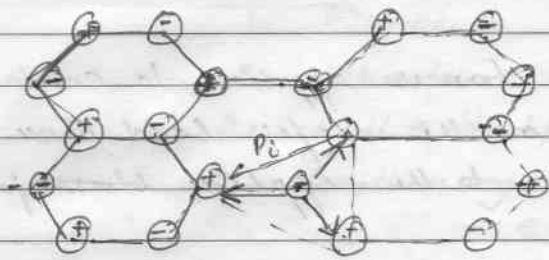
↓ external stress



$$\sum \vec{P}_i \neq 0$$

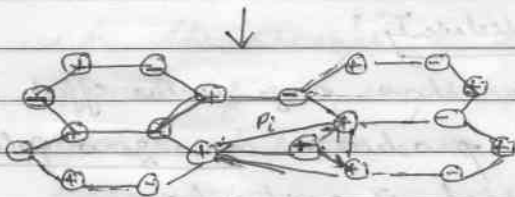
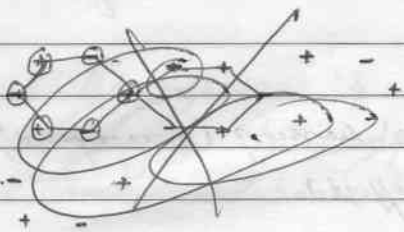
} crystal with center of symmetry

Reference: Prof. Thanhong Gu; MEMS Principles and Scaling Law: (lecture ppt).



$$\vec{\epsilon} P_i = 0$$

Crystals without
center of
symmetry.



$$\vec{\epsilon} P_i \neq 0$$

↑ external stress.

(Fig. reference Prof. Ninghong Gu, *NESS principles and scaling law, Lecture ppt*)

i.e. the piezoelectricity is the linear coupling between polarization (P_i) and the applied stress (σ_{ij}) or ~~stress~~ strain (ϵ_{ij}).

$$P_i = d_{ijk} \sigma_{jk}$$

$$P_i = e_{ijk} E_k$$

Both d_{ijk} and e_{ijk} are piezoelectric charge coefficients. The converse piezoelectric effect yields a mechanical response resulting from an applied electric field:

$$\epsilon_{ij} = d_{ij} E_k$$

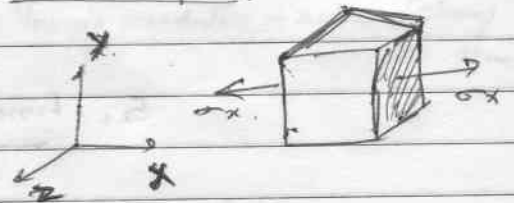
$$\sigma_{ij} = e_{kij} E_k$$

Hence piezoelectric materials can be used either as strain/stress sensors or as actuators in which displacement is induced by an applied electric field (E_k).

Some definitions to understand the above concepts:-

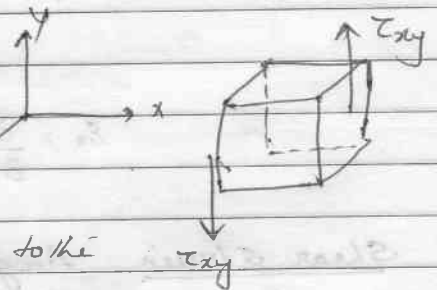
Stress: \rightarrow force per unit area.

Compressive $\sigma < 0$
Tensile $\sigma > 0$



Shear stress: Stress arising from the force vector component parallel to the cross-section.

(Here the area is the cross-sectional area parallel to the applied force vector).



Strain: is the relative change in shape or size of an object due to external forces. Since relative change no units.

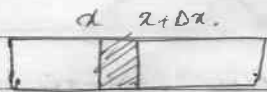
~~From~~ ~~Hook's~~ ~~law~~. we have

stress directly proportional to strain

Normal strain: fractional change in length

i.e. $\epsilon > 0 \rightarrow$ longer

$\epsilon < 0 \rightarrow$ shorter



Initial length = Δx



$x + u_x(x)$ $x + \Delta x + u_x(x + \Delta x)$

$$\begin{aligned} \text{Final length} &= (x + \Delta x + u_x(x + \Delta x)) - (x + u_x(x)) \\ &= \Delta x + u_x(x + \Delta x) - u_x(x) \end{aligned}$$

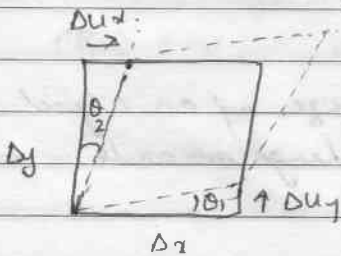
$$\epsilon_x = \frac{\text{Final length} - \text{Initial length}}{\text{Initial length}}$$

$$= \frac{u_x(x + \Delta x) - u_x(x)}{\Delta x} = \frac{\partial u_x}{\partial x}$$

$$\epsilon_x = \frac{\partial u_x}{\partial x}$$

Shear strain: Angle change due to shear stress.

: Represented as change in angle in radians.



$$\text{Shear strain} = \gamma_{xy} = \left[\frac{\Delta u_x}{\Delta y} + \frac{\Delta u_y}{\Delta x} \right]$$

$$= \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}$$

$$\gamma_{xy} = \theta_2 + \theta_1$$

Relation Between Stress and Strain:

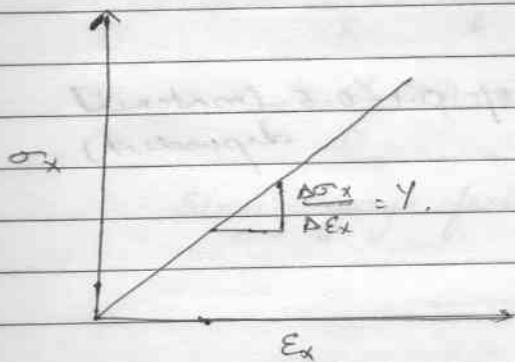
→ Depends on the regime of operation.

Linear regime Hooke's Law.

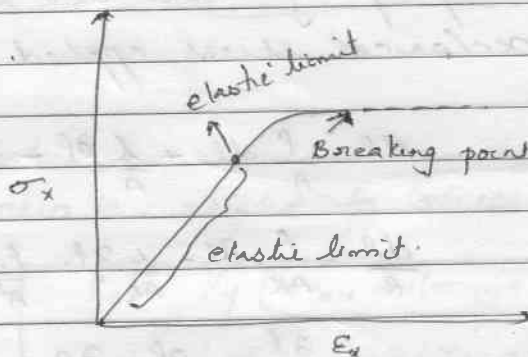
i.e. Stress is proportional to strain.

$$\sigma \propto \epsilon$$

i.e. $\sigma = Y\epsilon$, where Y → the proportionality constant known as Young's modulus = ratio of stress/strain



Range of Validity:



For larger stress, the material loses its elastic property and breaks. at this point Hooke's law is not valid.

{ Elastic property → deformation is recovered when the applied force is removed. }

Constitutive Equations:

Poisson's ratio:

- the negative ratio of transverse to axial strain.

When a material is compressed in one direction, it usually tends to expand in the other two directions perpendicular to the direction of compression. This is known as Poisson effect.

Poisson's ratio is a measure of this effect - i.e. ^{ratio of} fraction of expansion to the fraction of compression.

Poisson ratio in the range of 0.1 - 0.5 (material dependent).

$$\epsilon_y = -\nu \epsilon_x$$

Piezoresistivity

→ material property where bulk resistivity is influenced by mechanical stress applied to material.

$$\rightarrow R = \frac{\rho l}{A} \quad dR = \frac{\rho}{A} dl + \frac{l}{A} d\rho - \frac{\rho l}{A^2} dA$$

$$\frac{dR}{R} = \frac{\rho}{AR} dl + \frac{l}{AR} d\rho - \frac{\rho l}{A^2 R} dA$$
$$= \frac{dl}{l} + \frac{d\rho}{\rho} - \frac{dA}{A}$$

Assume cylindrical geometry

$$A = \frac{\pi D^2}{4} \quad \frac{dA}{A} = \frac{2 dD}{D}$$

Now, from poisson ratio

$$\nu = - \frac{\partial D/D}{\partial L/L}$$

$$-\nu \cdot \frac{\partial L}{L} = \frac{\partial D/D}{\partial L/L}$$

$$\frac{\partial A}{A} = -2 \cdot \nu \cdot \frac{\partial L}{L}$$

$$\frac{\partial R}{R} = \frac{\partial L}{L} + \frac{\partial P}{P} + 2 \cdot \nu \cdot \frac{\partial L}{L}$$

$$\frac{\partial R}{R} = (1+2\nu) \frac{\partial L}{L} + \frac{\partial P}{P}$$

Strain gauge factor: $GF = \frac{\partial R/R}{\partial L/L}$
 $= (1+2\nu) + \frac{\partial P/P}{\partial L/L}$

$$GF = (1+2\nu) + \frac{\partial P/P}{\epsilon}$$