Lumped elements modeling

Consider the case of a cantilever beam acted upon by an external force $\bar{F}$ at one end.

Q: How to solve?

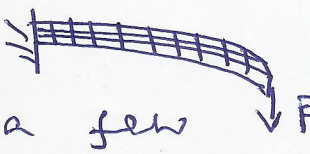
Ans. Three possible methods include:

(A.) Solve the elastic model with the suitable boundary conditions for the continuous beam.

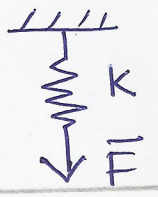
Analytic

(B.) Discretize the beam into a large number of sub-elements. Solution of PDE's in these sub-elements is trivial (numerically solve on computer).

FEM



(C.) Discretize the problem into a few lumped elements that captures the (essential) important characteristics.



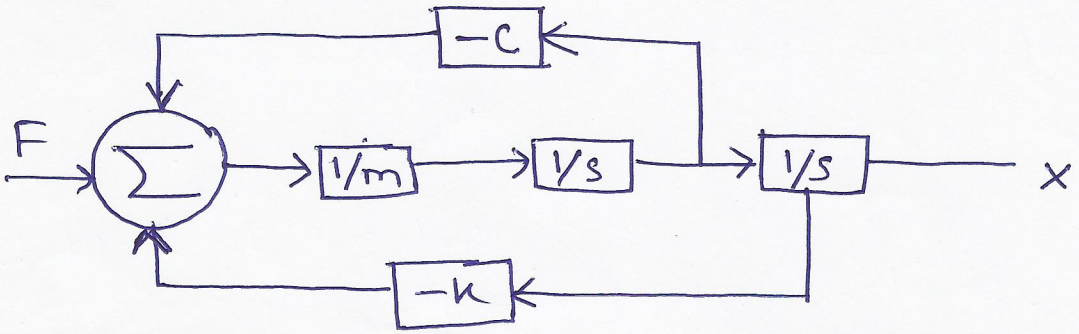
Lumped Elements modeling

	Advantages	Disadvantages
Analytic	Complete solution	Rare problems have analytic solutions
FEM	Heavy on details / Exact	• Heavy on resources • Obscure details
LEM	Essential details ("gross") Easy for designers	Miss out finer details

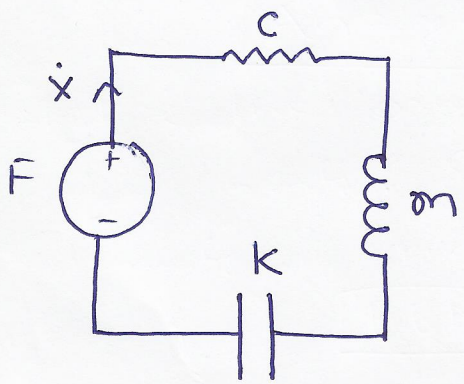
LEM

→ Goal: Understand the equivalence

Block representation vs circuit representation.

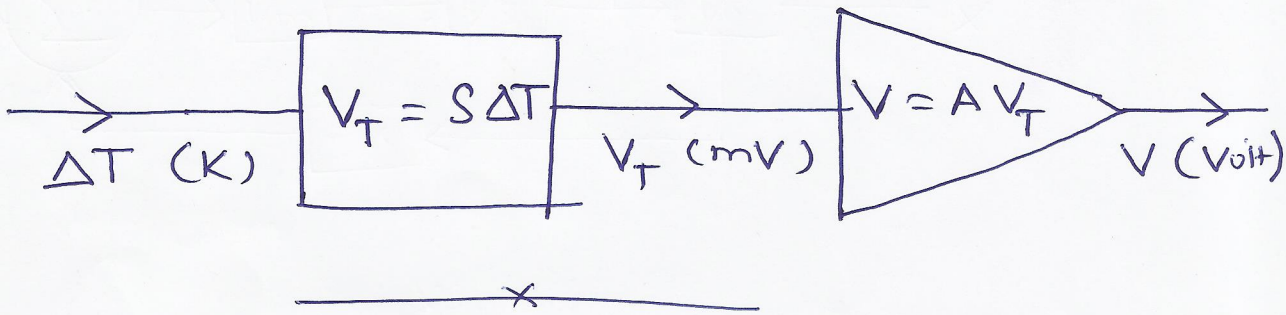


vs



A short primer: LEM

Example: Open loop temp. meas. system using block diagram.



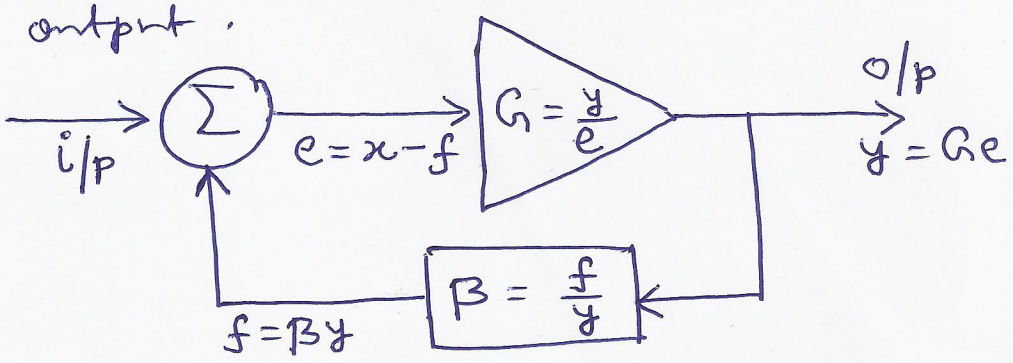
Open loop: measurement system. → Measure a variable

Closed loop: control system (prefered)

→ maintain a variable (e.g., T, v, direction, ...) to a desired value.

- sensor → to measure
- actuator → to regulate

Typical control system: output of the system is fed back to the input where the error signal is sent along forward path to attain desired output.



$$y = Ge$$

$$e = x - f$$

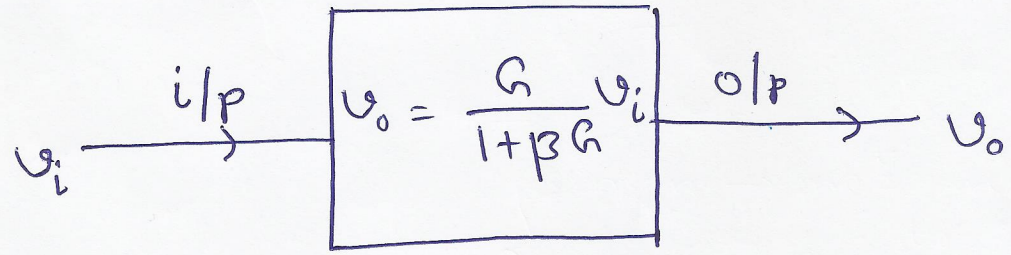
$$f = \beta y$$

$$\Rightarrow y = G(x - f) = G(x - \beta y)$$

$$\Rightarrow y(1 + \beta G) = Gx$$

$$\therefore \frac{y}{x} = \frac{G}{1 + \beta G}$$

∴ the block representation can be simplified to,



Note:
Though simple, details regarding sensor is obfuscated.

Typical example

Iris controller. (Details in class)