Continuum mechanical version of models for MEMS & NEMS.

{Ref: Modeling MEMS & NEMS by J. A. Pelesko & D. H. Bernstein Chapman & Hall / CRC.

→ Molecular details are ignored & components are treated in the continuum limit.

→ MEMS & NEMS comprise of mechanical beams, suspensions, membranes, guides, anchors, etc.

→ Functions such as density, temperature, velocity, displacement are assumed to be smoothly varying continuous functions of position.

→ continuum theory is good provided the length scales of interest are much longer than the length scales of molecular variation in the system under consideration.

(continuum hypothesis).

Lin & Segel thought experiment

Suppose we are interested in defining density of a fluid at a point in space at a given moment of time:

$$\rho(x, y, z, t).$$

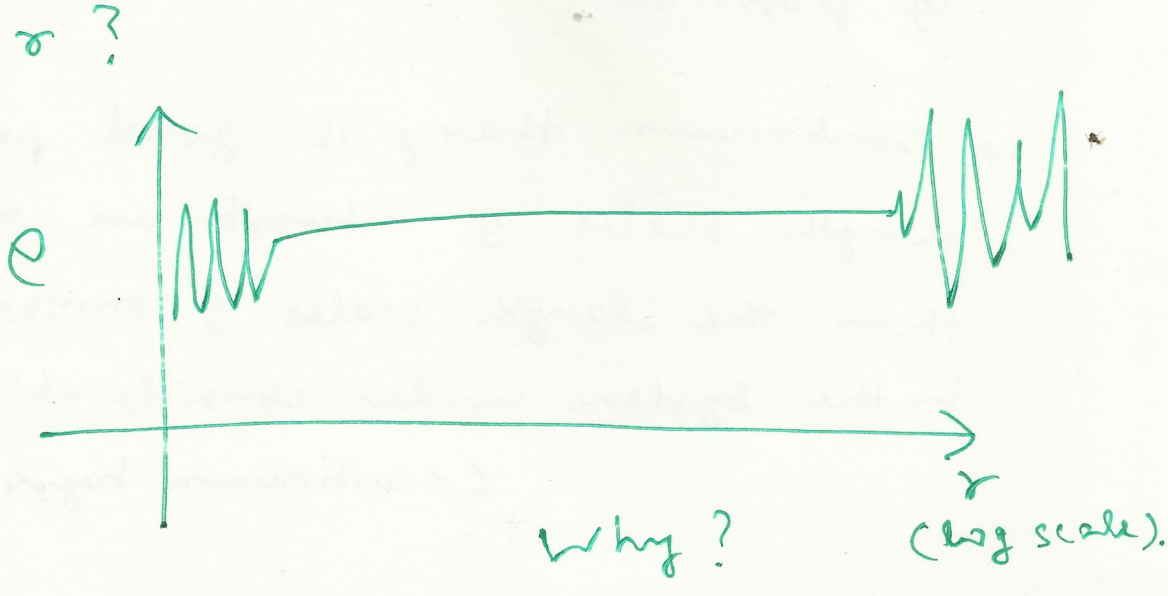
Imagine drawing a "little" ball of radius r around (x, y, z).

Measure mass M contained in it.

$$\therefore \rho = \frac{M}{\frac{4}{3}\pi r^3}.$$

How does $\rho(x, y, z, t)$ depends on r?

Ans:-



Why?

Fundamental continuum theories:

- Heat transfer
- Elasticity
- Fluid dynamics
- Electromagnetism

(A.) Heat transfer

Consider a volume V inside a material of interest.

In absence of internal sources & sinks of heat, conservation of energy

$\Rightarrow$ Change in energy $\equiv$ Flux of energy through surface

$$\frac{d}{dt}\int_V \rho C_p T \, dV.$$

Let flux of heat into the body is described by the vector $\vec{Q}$.

$\therefore$ Flux through the surface is given by the surface integral over the normal component of $\vec{Q}$,

$$\equiv -\int_S \bar{Q} \cdot \hat{n}\, ds$$

$$\Rightarrow \frac{d}{dt}\int_V \rho C_p T\, dV = -\int_S \bar{Q} \cdot \hat{n}\, ds$$

$$= -\int_V \nabla \cdot \vec{Q}\, dV$$

(using divergence theorem).

$$\Rightarrow \int_V \left(\rho C_p \frac{\partial T}{\partial t} + \nabla \cdot \bar{Q} \right) dV = 0.$$

Since, choice of dV is arbitrary,

$$\Rightarrow \boxed{\rho C_p \frac{\partial T}{\partial t} + \nabla \cdot \bar{Q} = 0.}$$

Fourier's assumption:

$$\vec{Q} = -K\nabla T.$$

K is thermal conductivity of the medium.
(validity at micro/nano scale !!).

$$\Rightarrow \boxed{\rho C_p \frac{\partial T}{\partial t} = \nabla \cdot K \nabla T}$$

valid for an <u>anisotropic</u> inhomogeneous medium.

K a tensor

ρ, C_p, K are dependent on (x, y, z)

For isotropic homogeneous medium,

$$\frac{\partial T}{\partial t} = \left(\frac{K}{\rho C_p}\right)\nabla^2 T.$$

$$\frac{K}{\rho C_p} = \mathbb{K}, \text{ thermal diffusivity.}$$

Initial & Boundary conditions

Initial condition

$$T(x, y, z, 0) = f(x, y, z).$$

(specify temp. of the body at every point in space at $t = 0$).

Boundary condition

(i) Dirichlet B.C

$$T\Big|_S = T_A$$

temperature is specified on the boundary of the body.

Physically $\Rightarrow$ contact with a constant temperature bath.

(ii) Neumann B.C.

$$K \nabla T \cdot \hat{n}\Big|_S = q$$

Specifies flux of heat through the boundary.

(iii) $q = 0$ (perfectly insulated body).

$$\Rightarrow \; K \, \nabla T \cdot \hat{n} \Big|_S = 0.$$

(iv) $$K \, \nabla T \cdot \hat{n} \Big|_S = -h(T - T_A) \Big|_S$$

(Newton cooling)

"Forced convection" + lot of things

(v) $$K \, \nabla T \cdot \hat{n} \Big|_S = \sigma e (T^4 - T_A^4) \Big|_S .$$

Familiar inputs required during simulation using packages like COMSOL, etc.

σ = Stefan-Boltzmann constant.

e = emissivity of the surface $\{ \in [0, 1]$.

closeness to black body.

Example

Isotropic homogeneous sphere of radius a (μm) initially at a temperature T_0 immersed in a constant temperature bath of temperature T_A.

- Spherically symmetric
- $T(t) \equiv T(r,t)$.

$$\Rightarrow \rho C_p \frac{\partial T}{\partial t} = K \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial T}{\partial r}.$$

with initial condition

$$T(r,0) = T_0.$$

& boundary condition

$$T(a,t) = T_A.$$

Mathematically, T is required to be bounded at $r=0$.

$$\Rightarrow |T(0,t)| < \infty.$$

Let $u(r,t) = r\,T(r,t)$.

$$\Rightarrow \rho C_p \frac{\partial u}{\partial t} = K \frac{\partial^2 u}{\partial r^2}$$

$$u(r,0) = r T_0.$$

$$u(a,t) = a T_A.$$

$$u(0,t) = 0.$$

Solving,

$$\Rightarrow T(r,t) = T_A - \frac{2}{\pi r}\sum_{n=1}^{\infty}\frac{1}{n}\left\{ a T_A(-1)^n + T_0\left((-1)^n - 1\right)\right\} e^{-\frac{n^2\pi^2}{a^2}\kappa t} \sin\left(\frac{n\pi r}{a}\right)$$

κ is thermal diffusivity.

Problem:

Assume the sphere is made of Si.

Using www find κ.

Let $T_0 = 1000K$ & $T_A = 300K$.

If the radius is $10\mu m$, how long does it take for the center of the sphere to cool to 90% of its steady state value?

Repeat for $a = 100nm, 1\mu m, 100\mu m, 1cm$.