Can we do something about reducing the number of components?

Use:

"A physical property measured along two different directions must be equal if these two directions are crystallo-graphically equivalent".

Example:

Let $J(x, y, z)$ & $J'(x', y', z')$ represent the same quantity.

$$\begin{pmatrix} J_1' \\ J_2' \\ J_3' \end{pmatrix} = \underbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}}_{\text{Unitary matrix}} \begin{pmatrix} J_1 \\ J_2 \\ J_3 \end{pmatrix}.$$

$A^{-1} = A^T$.

e.g., for centrosymmetry, $A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$.

for rotation about principal axis $A = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

Electric field is transformed in same way; $\Rightarrow$

$$\begin{pmatrix} E_1' \\ E_2' \\ E_3' \end{pmatrix} = A \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix}$$

or, $E_i' = a_{ij} E_j$.

The we can calculate $\sigma'$ tensor defined by $J' = \sigma' E'$.

$$\Rightarrow \begin{pmatrix} \sigma_{11}' & \sigma_{12}' & \sigma_{13}' \\ \sigma_{21}' & \sigma_{22}' & \sigma_{23}' \\ \sigma_{31}' & \sigma_{32}' & \sigma_{33}' \end{pmatrix} = A \sigma A^{-1} = A \sigma A^{T}.$$

or, $\boxed{\sigma_{ij}' = a_{ik} a_{jl} \sigma_{kl}.}$ — (vii)

Example: When the crystal has a 2-fold axis along the z-axis, the elecrical conductivity should have same tensorial from in terms of transformation $\underbrace{\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\tau_2}$.

i.e., $\sigma' = \tau_2 \sigma \tau_2^{-1}$

"After little algebra" $\Rightarrow$ $\sigma_{31} = \sigma_{13} = \sigma_{32} = \sigma_{23} = 0$. (Note $\varphi = \frac{2\pi}{\text{"2"}}$ ).