PH101 (Physics-I) Tutorial-I (July 28, 2014)

[Dimensional Analysis, Approximation Methods and Vectors]

1. (a) Consider a vibrating water drop, whose frequency depends on its radius R, mass density ρ, and surface tension S. The units of surface tension are (force)/(length). How does ν depend on R, ρ, and S?
(b) How does the speed of waves in a fluid depend on its density, ρ, and bulk modulus, B (which has units of pressure, which is force per area)?

2. Consider the Atwoods machine (shown in the figure), consisting of three masses and three frictionless pulleys. It can be shown that the acceleration of m_1 is given by: $a_1 = g\,\frac{3m_2\,m_3 - m_1(4\,m_3+m_2)}{m_2\,m_3+m_1(4\,m_3+m_2)}$, with upward taken to be positive (we will talk more about it later in the course).
Find a_1 in the following special cases:

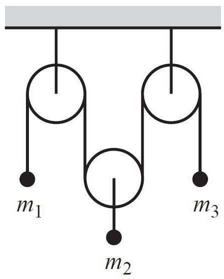

Figure 1: An Atwood's machine.

(a) $m_2 = 2\,m_1 = 2\,m_3$.
(b) m_1 much larger than both m_2 and m_3.
(c) m_1 much smaller than both m_2 and m_3.
(d) $m_2 \gg m_1 = m_3$.
(e) $m_1 = m_2 = m_3$.

3. Find the angle between any two diagonals of a cube.

4. The trajectory of a charged particle moving in a magnetic field is given by

$$\vec{r} = b\,cos(\Omega t)\hat{i} + b\,sin(\Omega t)\hat{j} + c\,t\hat{k} \quad (1)$$

where b, Ω and c are positive constants. Show that the particle moves with constant speed and find the magnitude of its acceleration.

5. As shown in Figure 2, the cycloid can be parametrized as $x = a(\theta - sin\theta)$, $y = a(1 - cos\theta)$, $z = \theta$, where, $0 < \theta < 2\pi$.
(a) Find the unit tangent vector to the cycloid at the point with parameter θ.
(b) Also obtain the unit normal vector and the curvature of the cycloid.

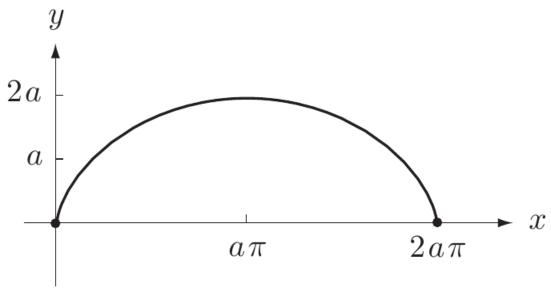

Figure 2: A cycloid.

6. Let $\vec{A}$ be an arbitrary vector and let $\hat{n}$ be the unit vector in a certain fixed direction. Show that $\vec{A} = (\vec{A}.\hat{n})\hat{n} + (\hat{n} \times \vec{A}) \times \hat{n}$

7. Making use of the Taylor series expansion, verify the following identity:
$exp(ix) = cos(x) + isin(x)$