PH101 (Physics-I) **Tutorial-IV (August 18, 2014)**

[Simple Harmonic Motion]*

1. Verify if the following forces are conservative:
   (a) $\vec{F_1} = -2x\hat{i} - 2y\hat{j} - 2z\hat{k}$
   (b) $\vec{F_1} = y\hat{i} - x\hat{j}$

2. A particle of mass 2 kg moves on the positive z-axis under the force field $\vec{F} = \left(\frac{4}{x^2} - 1\right)\hat{i}$. Initially the particle is released from rest at the point $x = 4$ m. Find the extreme points and the period of the motion.

3. A particle of mass M moves under the influence of a potential $V(x) = \frac{a}{x^2} - \frac{b}{x}$, where a, $b > 0$. Obtain the equilibrium point and the frequency of small oscillations about that point.

4. A damped oscillator (with $m\ddot{x} = -kx - b\dot{x}$) has initial position x_0 and speed v_0. After a long time, the mass m will come back to rest at the origin. Obtain the work done by the damping force.

5. (a) Show that an overdamped or critically damped oscillator can cross the origin at most once.
   (b) A critically damped oscillator with natural frequency Ω starts out at position $x_0 > 0$. What is the maximum initial speed (directed toward the origin) it can have and not cross the origin?

6. For a damped harmonic oscillator, $m\ddot{x} = -\alpha x - \beta\dot{x}$, or alternatively, $\ddot{x} + 2K\dot{x} + \Omega^2 x = 0$, where, $\alpha = m\Omega^2$ and $\beta = 2mK$. Show that $\frac{dU}{dt} = -2mK\dot{x}^2$, where, U is the total energy.

*Note: Please follow the strategies for "Problem Solving" explained in the class.