भारतीय प्रौद्योगिकी संस्थान पटना

INDIAN INSTITUTE OF TECHNOLOGY PATNA

MID-SEMESTER EXAMINATION

PH103 (PHYSICS-I) September 24, 2018

INSTRUCTIONS

- Answers to all parts of a given question must be answered together.
- Questions 6-10 carry one mark each and the answers to these should be written in a single page.
- Rough work should be shown separately.

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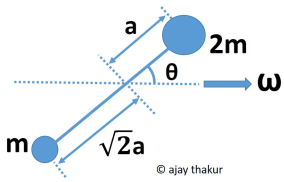
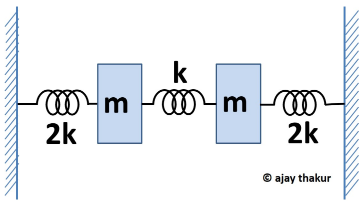
1. For $\vec{F} = 3m\dot{r}\dot{\theta}\hat{\theta}$, show that, $\dot{r} = \pm\sqrt{Ar^4 + B}$, where A and B are arbitrary constants. Derive the necessary expressions as per your requirement. **[5]**

2. A dumb-bell shaped rigid body is rotating with angular frequency ω about an axis as shown in the figure below. Considering, the rigid rod connecting the two masses to be massless, obtain the torque. (Hint: Choose suitable principal axes and use Euler's equations for rigid body dynamics.) **[5]**



3. (a) For a coupled oscillator shown in the figure below, write down the equations of motion under the conditions of small deflections and no damping.
(b) Obtain the corresponding equations in a matrix form and write down the expressions for the mass and stiffness matrices $\bar{M}$ and $\bar{\mathcal{K}}$, respectively.
(c) Obtain the eigenvalues and eigenvectors and interpret the results.
(d) Show that $\Sigma_{i=1}^{2} \mid e_i >< e_i \mid = \bar{I}$, where, $\bar{I}$ is the 2×2 identity matrix and $\mid e_i >$ are the eigenvectors in the 'bra-ket' notation. **[5]**



4. Consider a driven damped harmonic oscillator (a spring-mass-damper system) with the following parameters: $m = 0.1\,\text{kg}$, $k = 0.3\,\text{N/m}$, and, $\beta = 0.4\,\text{Ns/m}$ (symbols have their usual meanings such that the damping force is $-\beta\frac{dx}{dt}$). An external driving force given by $cos(2t)$ acts on the spring-mass-damper system. Initial conditions are as follows: at $t = 0$, $x(0) = 0$ and $\dot{x}(0) = 1\,\text{m/s}$.
(a) Construct the differential equation for the system.
(b) Solve for $x(t)$. Find both the complimentary function (solution to homogeneous part which determines the transient response) and the particular integral. Use initial conditions to determine the constants of integration appearing in the complete solution. **[5]**

5. Prove that for an attractive central force which varies as inverse square of radial distance, the trajectories are conic sections. [**5**]

6. The nature of constraint in the case of a simple pendulum with a rigid support (with no interactions with ambient air) is:

   (a) scleronomic, holonomic, bilateral and conservative
   (b) rheonomic, holonomic, bilateral and conservative
   (c) scleronomic, non-holonomic, bilateral and conservative
   (d) scleronomic, holonomic, unilateral and conservative
   (e) scleronomic, holonomic, bilateral and non-conservative

7. The dependence on Planck's constant ($\hbar$), speed of light (c), and, the universal gravitational constant (G) of Planck's time and Planck's length (from a dimensional perspective) are:

   (a) $\sqrt{\frac{\hbar c}{G}}$ and $\sqrt{\frac{\hbar G}{c^5}}$, respectively.
   (b) $\sqrt{\frac{\hbar G}{c^5}}$ and $\sqrt{\frac{\hbar c}{G}}$, respectively.
   (c) $\sqrt{\frac{\hbar G}{c^5}}$ and $\sqrt{\frac{\hbar G}{c^3}}$, respectively.
   (d) $\sqrt{\frac{\hbar G}{c^3}}$ and $\sqrt{\frac{\hbar G}{c^5}}$, respectively.
   (e) $\sqrt{\frac{\hbar G}{c^3}}$ and $\sqrt{\frac{\hbar c}{G}}$, respectively.

8. Rotation of a rigid body about one of its principal axis ($\hat{1}$) is stable if:

   (a) $I_1 < I_2 < I_3$ or, $I_3 < I_2 < I_1$
   (b) $I_2 < I_1 < I_3$ or, $I_3 < I_1 < I_2$
   (c) $I_1 < I_2 < I_3$ or, $I_2 < I_1 < I_3$
   (d) $I_3 < I_2 < I_1$ or, $I_3 < I_1 < I_2$
   (e) $I_3 < I_2 < I_1$ or, $I_2 < I_1 < I_3$

9. A high speed hydrofoil races across the ocean at the equator at a speed of 320 km/hr northward. The fractional change in gravity $\frac{\Delta g}{g}$ measured by a passenger on the hydrofoil due to Coriolis force is:
   (a) -1.33×10^{-3}, (b) 1.33×10^{-3}, (c) -2.33×10^{-3}, (d) 2.33×10^{-3}, (e) 0

10. Which of the following relationship between a fixed frame and the rotating frame is true?
    (a) $\frac{d}{dt}|_{fix} = -\frac{d}{dt}|_{rot} + \vec{\omega}\times$, (b) $\frac{d}{dt}|_{fix} = \frac{d}{dt}|_{rot} - \vec{\omega}\times$
    (c) $\frac{d}{dt}|_{fix} = -\frac{d}{dt}|_{rot} - \vec{\omega}\times$, (d) $\frac{d}{dt}|_{fix} = \frac{d}{dt}|_{rot} + \vec{\omega}\times$