PH103 (Physics-I) **Tutorial-III (August 30, 2018)**

1. A projectile of mass m is fired from the origin at speed v_0 and angle θ. It is attached to the origin by a spring with spring constant k and relaxed length zero.
(a) Find $x(t)$ and $y(t)$.
(b) Verify that for small $\omega \equiv \sqrt{\frac{k}{m}}$, the trajectory reduces to normal projectile motion.
(c) Verify that for large ω, the trajectory reduces to simple harmonic motion, *i.e.*, oscillatory motion along a line (at least before the projectile smashes back into the ground!).
(d) Physically interpret "small " and "large ".
(e) What value should ω take so that the projectile hits the ground when it is moving straight downward?

2. **Alternate derivation of T**: For small oscillations, the period of a pendulum is approximately $T \approx 2\pi\sqrt{\frac{l}{g}}$ independent of the amplitude, θ_0. In class we used a perturbative approach for estimating the corrections to T when amplitude θ_0 becomes large. In this tutorial problem, an alternate method for solving the same problem is illustrated.
(a) Using $dt = \frac{dx}{v}$, show that the exact expression for T is

$$T = \sqrt{\frac{8l}{g}} \int_0^{\theta_0} \frac{\mathrm{d}\theta}{\sqrt{\cos\theta - \cos\theta_0}}$$

(b) Making use of the identity $\cos\phi = 1 - 2\sin^2\frac{\phi}{2}$, write T in terms of sines [why!]. Make a suitable change of variables,

$$\sin x \equiv \frac{\sin\frac{\theta}{2}}{\sin\frac{\theta_0}{2}}$$

Now expand the integrand in powers of θ_0 and evaluate the resulting integrals to show that

$$T \approx 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{\theta_0^2}{16} + \cdots\right)$$

3. **Alternate derivation of $z_{\max}$**: In class we worked out the problem where an object of mass m was thrown upwards with an initial speed u and in between obtained an expression for the time t required to attain a velocity v. There was a drag force due to the surrounding atmosphere which was proportional to the mass and velocity of the object with κ as the proportionality constant). Using expression for t, show that:
$v = ue^{-\kappa t} - \frac{g}{\kappa}(1 - e^{-\kappa t})$.

Starting from expression for v thus obtained, find z_{max} (the maximum height attained by the object).

4. A body is released from rest and moves under uniform gravity in a medium that exerts a resistance force proportional to the square of its speed and in which the bodys terminal speed is V_{T}. Show that the time taken for the body to fall a distance H is $\frac{V_{\mathrm{T}}}{g} cosh^{-1}(e^{\frac{gH}{V_{\mathrm{T}}^2}})$.

5. A ball is thrown with speed v_0 at an angle θ. Let the drag force from the air take the form $F_{\mathrm{d}} = -\beta v = -m\alpha v$. (a) Find $x(t)$ and $y(t)$. (b) Assume that the drag coefficient takes the value that makes the magnitude of the initial drag force equal to the weight of the ball. If your goal is to have x be as large as possible when y achieves its maximum value, show that θ should satisfy $sin\theta = \frac{\sqrt{5}-1}{2}$ (inverse of the golden mean!).