PH103 (Physics-I) **Tutorial-IV (September 6, 2018)**

[**Note: Please ensure to complete leftover problems from Tutorial-III**]

1. For a damped harmonic oscillator, $m\ddot{x} = -kx - \beta\dot{x}$, or alternatively, $\ddot{x} + 2\Gamma\dot{x} + \omega_0^2 x = 0$, where, $k = m\omega_0^2$ and $\beta = 2m\Gamma$. Show that $\frac{dU}{dt} = -2m\Gamma\dot{x}^2$, where, U is the total energy.

2. A damped oscillator (with $m\ddot{x} = -kx - \beta\dot{x}$) has initial position x_0 and speed v_0. After a long time, the mass m will come back to rest at the origin. Obtain the work done by the damping force.

3. (a) Show that an overdamped or critically damped oscillator can cross the origin at most once.
   (b) A critically damped oscillator with natural frequency ω_0 starts out at position $x_0 > 0$. What is the maximum initial speed (directed toward the origin) it can have and not cross the origin?