भारतीय प्रौद्योगिकी संस्थान पटना
INDIAN INSTITUTE OF TECHNOLOGY PATNA

PH103 (Physics-I) **Tutorial-VII (October 18, 2018)**

[Quantum Mechanics]

1. Consider a particle whose normalized wave function is given by $\Psi(x) = 2\alpha\sqrt{\alpha}x\, e^{-\alpha x}$ for $x > 0$ and zero otherwise.

   (a) Obtain the probability density and find the value of x for which it is maximum.

   (b) Calculate the expectation values $< x >$ and $< x^2 >$.

   (c) Estimate the probability for the particle to be found between $x = 0$ and $x = 1/\alpha$.

   (d) Obtain the corresponding momentum space wave function $\phi(p_x)$. Find the expectation values $< p_x >$ and $< p_x^2 >$.

   (e) Using the above results, estimate Δx and Δp_x, and show that $\Delta x \cdot \Delta p_x \geq \hbar/2$.

2. Consider the probability density of a normalized Gaussian wave function: $\rho(x) = Ae^{-\lambda(x-a)^2}$, where, A, a, and λ are constants.

   (a) Determine A.

   (b) Obtain $< x >$, $< x^2 >$ and Δx.

   (c) Sketch $\rho(x)$ and interpret the results.

3. A particle of mass m is in the state $\psi(x,t) = Ae^{-a[(mx^2/\hbar)+it]}$, where, A and a are positive real constants.

   (a) Obtain A.

   (b) Find the potential energy function $V(x)$ for which $\psi(x,t)$ satisfies the corresponding Schrödinger equation.