भारतीय प्रौद्योगिकी संस्थान पटना

INDIAN INSTITUTE OF TECHNOLOGY PATNA

PH103 (Physics-I) **Tutorial-VIII (October 25, 2018)**

[Note: The following tutorial questions are based primarily on unsolved problems from the excellent textbook on 'Introduction to Quantum Mechanics' by David J. Griffiths. Students are recommended to read the book.]

1. (a) If the wavefunction corresponding to $E = 0$ is given by $\psi(x) = \frac{A}{x^2+a^2}$, (where, a is a constant and A is the normalization factor), obtain the corresponding potential $V(x)$.
   (b) The wavefunction for a potential $V(x) = \alpha^2 x^2$ is given by $\psi(x) = exp(-\sqrt{\frac{m\alpha^2}{2\hbar^2}}x^2)$. Obtain the corresponding energy eigenvalue.

2. For a particle constrained to move in $0 \leq x \leq L$, the wavefunction is given by $\psi(x) = \sqrt{\frac{2}{L}} sin(\frac{n\pi x}{L})$. Obtain $< p_x >$ and $< p_x^2 >$.

3. In class we proved the first Ehrenfest's theorem ($\frac{d<x>}{dt} = \frac{<p_x>}{m}$). Prove Ehrenfest's second theorem: $\frac{d<p_x>}{dt} =< -\frac{\partial V}{\partial x} >$.

4. $P_{ab}(t)$ is the probability of finding a quantum mechanical object in the range $a \leq x \leq b$ at a time t. Show that $\frac{dP_{ab}}{dt} = J(a,t) - J(b,t)$. Here J is the probability current as defined in class [Hint: Use continuity equation connecting probability density and probability current density as discussed in the class.]

5. Show that $\frac{d}{dt}\int_{-\infty}^{\infty} \Psi_1^* \Psi_2 dx = 0$, for any two (normalizable) solutions to the Schrödinger equation, Ψ_1 and Ψ_2.