Newton's laws of motion.

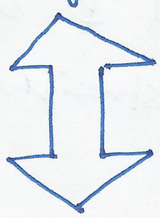
First law: When all external influences on a particle are removed, the particle moves with constant velocity.
(law of inertia)

Second law: When a force $\vec{F}$ acts on a particle of mass m, the particle moves with instantaneous acceleration $\vec{a}$ given by the formula,

$$\vec{F} = m\vec{a},$$

Note: Unit of force is implied by the units of mass & acceleration.
(definition of force)

Third law: When two particles exhert forces upon each other, these forces are:
(i) equal in magnitude, (ii) opposite in direction, & (iii) parallel to the straight line joining the two particles.
(every action has equal & opposite reaction)



Q1. In which frame of reference are the laws true?

Q2. What are the definitions of mass & force?

Alternate definition

First law

"Every body continues to be in a state of rest or of uniform motion in a straight line unless it is compelled to change that state by external forces acting on it".

Second law

"The time rate of change of momentum of a particle is proportional to the external force & is in the direction of force".

Third law

"To every action, there is always ~~a~~ an equal & opposite reaction"

i.e.,

"The mutual action of any two bodies are always equal & oppositely directed along the same straight line".

What reference frame should we use?

1. Fixed ✗ { everything in the Universe is moving w.r.t something else!

2. Inertial frame ✓

A reference frame in which the Newtons First law is true.

If ∃ one inertial frame, then ∃ infinitely many, with each frame moving with constant velocity (& no rotation) relative to any other.

⇓

{ There exists in nature a unique class of mutually unaccelerated reference frames (the inertial frames) in ~~at~~ which the First law is true.

<u>Practical inertial frames</u>:

~~Earth~~
- Earth
- Geocentric
- Heliocentric

(* ∃ ≡ there exists.)

Law of mutual interaction

Suppose that two particles P_1 & P_2 interact with each other & that P_2 induces an instantaneous acceleration $\vec{a}_{12}$ in P_1, while P_1 induces an instantaneous acceleration $\vec{a}_{21}$ in P_2.

Then,

(i.) these accelerations are opposite in direction & ||lel to the straight line joining P_1 & P_2.

(ii.) the ratio of magnitudes of these accelerations, $\frac{|\vec{a}_{21}|}{|\vec{a}_{12}|}$ is a constant independent ~~of~~ of the nature of mutual interaction between P_1 & P_2, and independent of the positions & velocities of P_1 & P_2.

Law of multiple interactions

If the particles $P_0, P_1, \ldots, P_n$ are interacting with each other & that all other influences are removed. Then accl^n. induced in P_0 can be written as,

$$\vec{a}_0 = \vec{a}_{01} + \vec{a}_{02} + \ldots + \vec{a}_{0n}$$

where,

$a_{01}, a_{02}, \ldots, a_{0n}$ are the

accl^n. ~~when~~ that P_0 will have if the particles $P_1, P_2, \ldots$ were individually interacting with P_0.

$\Downarrow$

Experimental basis of Newton's laws

Law of inertia

~~Law~~.

+

Law of mutual interaction

+

Law of multiple interaction

$\Downarrow$

Validity of Newton's laws in any inertial frame of reference.

The law of gravitation

Gravitational forces that two particles of masses m_1 & m_2 exert upon each other have a magnitude

$$\frac{G m_1 m_2}{R^2}$$

where, R is the distance between the particles & G is a universal constant.

$$G = 6.67 \times 10^{-11} \ Nm^2/kg^2 .$$

Ref :-
- Kleppner & Kolenkow
- R. Douglas Gregory
- David Morin

Newton's 2nd law in usage

Consider the following physical scenario.

A particle is projected vertically upwards with initial speed u & moves in a vertical straight line under the influence of uniform gravity. Find maximum height reached (z_{max}) and time (t_{max}) required to do so in

Case I : No air drag.

Case II : Air drag proportional to velocity.

Comprehend the results (in light of Lecture 2).

Soln.

Case(I:) Newton's 2nd law

$$\Rightarrow \quad m\frac{dv}{dt} = -mg. \qquad \text{i.e., } \frac{dv}{dt} = -g.$$

$$\Rightarrow v = -gt + C.$$

At $t=0$, $v=u$.

$\therefore$ $C = u$.

Thus, $\boxed{v = u - gt}$ $\Rightarrow t_{max} = \frac{u}{g}$.

At $t=0$, $z=0$.

At $z = z_{max}$, $t = t_{max}$ & $v=0$.

$$\frac{dz}{dt} = u - gt \Rightarrow z = ut - \frac{1}{2}gt^2 + C_2.$$

At $t=0$, $z=0 \Rightarrow C_2 = 0$.

At $t = t_{max}$, $z = z_{max} \Rightarrow z_{max} = ut_{max} - \frac{1}{2}gt_{max}^2$.

$$\therefore z_{max} = \frac{u^2}{2g}.$$

Case (II.) Newton's law, $\Rightarrow m\frac{dv}{dt} = -mg - Kmv$.

$v(t) = \hat{u}$ at $t = 0$.

$v(t) = 0$ at $t = t_{max}$.

$z(t) = 0$ at $t = 0$.

$z(t) = z_{max}$ at $t = t_{max}$.

$$\int \frac{dv}{(g+Kv)} = -\int dt$$

$$\therefore -\frac{1}{K}\ln(g+Kv) = -t + C.$$

At $t = 0$, $v(t) = u$.

$$\Rightarrow C = -\frac{1}{K}\ln(g+Ku).$$

$$\Rightarrow \boxed{t = \frac{1}{K}\ln\left(\frac{g+Ku}{g+Kv}\right).}$$

At $t = t_{max}$, $v = 0$.

$$\therefore \boxed{t_{max} = \frac{1}{K}\ln\left(1 + \frac{Ku}{g}\right).}$$

invert

$$v = ue^{-Kt} - \frac{g}{K}(1 - e^{-Kt}).$$

As $t \to \infty$, $|v| = \frac{g}{K}$; $v = -\frac{g}{K}$.
(terminal velocity).

Check :- Terminal velocity $\Rightarrow \frac{dv}{dt} = 0$.
$\therefore$ From initial eq. $-mg - mKv = 0$.
$\therefore v_{term} = -\frac{g}{K}$.

Now $\frac{dv}{dt} = \frac{dv}{dz}\frac{dz}{dt} = v\frac{dv}{dz} = -g - Kv$.

$$\therefore -\int dz = \int \frac{v\,dv}{(g+Kv)}$$

$$= \frac{1}{K}\int\left(1 - \frac{g}{g+Kv}\right)dv.$$

$$= \frac{v}{K} - \frac{g}{K^2}\ln(g+Kv) + D.$$

When $z = 0$, $v = u$.

$$\Rightarrow D = -\frac{u}{K} + \frac{g}{K^2}\ln(g+Ku).$$

$$\Rightarrow z = -\frac{v}{K} + \frac{g}{K^2}\ln(g+Kv) + \frac{u}{K} - \frac{g}{K^2}\ln(g+Ku).$$

$$\therefore z = \frac{1}{K}(u-v) - \frac{g}{K^2}\ln\left(\frac{g+Ku}{g+Kv}\right).$$

When, $z = z_{max}$, then $v = 0$.

$$\Rightarrow \boxed{z_{max} = \frac{u}{K} - \frac{g}{K^2}\ln\left(1 + \frac{Ku}{g}\right)}.$$

Summary.

	Case I	Case II
t_{max}	$\frac{u}{g}$	$\frac{1}{k}\ln\left(1+\frac{ku}{g}\right)$
z_{max}	$\frac{u^2}{2g}$	$\frac{u}{k}-\frac{g}{k^2}\ln\left(1+\frac{ku}{g}\right)$

$$[kmv] = [F] = MLT^{-2}.$$

$$\Rightarrow [k] = MLT^{-2}M^{-1}L^{-1}T = T^{-1}.$$

$$\therefore \left[\frac{1}{k}\right] = T. \checkmark$$ Ok.

$$\left[\frac{u}{k}\right] = L. \checkmark$$

Special case: $\frac{ku}{g}$ is small

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots$$

$$t_{max} = \frac{1}{k}\ln\left(1+\frac{ku}{g}\right) \approx \frac{1}{k}\left[\frac{ku}{g} - \frac{1}{2}\frac{k^2u^2}{g^2} + \frac{1}{3}\frac{k^3u^3}{g^3} \cancel{+\cdots}\right]$$

$$\approx \frac{u}{g}\left[1 \underbrace{-\frac{1}{2}\left(\frac{ku}{g}\right) + \frac{1}{3}\left(\frac{ku}{g}\right)^2}_{\text{correction due to drag}} \cancel{+\cdots}\right].$$

upto 3rd term.

$$z_{max} = \frac{u}{k} - \frac{g}{k^2}\ln\left(1+\frac{ku}{g}\right) \approx \frac{u}{k} - \frac{g}{k^2}\left[\frac{ku}{g} - \frac{1}{2}\frac{k^2u^2}{g^2} + \frac{1}{3}\frac{k^3u^3}{g^3}\right]$$

$$\approx \frac{u^2}{2g}\left[1 \underbrace{-\frac{2}{3}\left(\frac{ku}{g}\right)}_{\text{correction due to linear drag}}\right].$$