PH101 (Physics-I) Tutorial-II (August 11, 2015)

[Newtons laws]*

1. For a smooth ("low jerk") ride, an elevator is programmed to start from rest and accelerate according to $a(t) = (a_m/2)[1 - cos(2\pi t/T)]$ for $0 \leq t \leq T$
$a(t) = -(a_m/2)[1 - cos(2\pi t/T)]$ for $T \leq t \leq 2T$
where a_m is the maximum acceleration and $2T$ is the total time for the trip. Draw sketches of $v(t)$, $a(t)$ and the jerk $j(t)$ as functions of time.

2. A body is released from rest and moves under uniform gravity in a medium that exerts a resistance force proportional to the square of its speed and in which the bodys terminal speed is V_T. Show that the time taken for the body to fall a distance H is $\frac{V_T}{g} cosh^{-1}(e^{\frac{gH}{V_T^2}})$.

3. A ball is thrown with speed v_0 at an angle θ. Let the drag force from the air take the form $F_d = -\beta v = -m\alpha v$. (a) Find $x(t)$ and $y(t)$. (b) Assume that the drag coefficient takes the value that makes the magnitude of the initial drag force equal to the weight of the ball. If your goal is to have x be as large as possible when y achieves its maximum value, show that θ should satisfy $sin\theta = \frac{\sqrt{5}-1}{2}$ (inverse of the golden mean!).

4. A particle is sliding along a smooth radial groove in a circular turntable which is rotating with constant angular speed Ω. The distance of the particle from the rotation axis at time t is observed to be $r = b\, cosh(\Omega t)$ for $t \geq 0$, where b is a positive constant. Find the speed of the particle (relative to a fixed reference frame) at time t, and also find the magnitude and direction of the acceleration.

5. The luckless Daniel (D) is thrown into a circular arena of radius a containing a lion (L). Initially the lion is at the centre O of the arena while Daniel is at the perimeter. Daniels strategy is to run with his maximum speed u around the perimeter. The lion responds by running at its maximum speed U in such a way that it remains on the (moving) radius OD. (i) Set up the differential equation satisfied by r (the distance of L from O). (ii) Find r as a function of t. (iii) If $U \geq u$, show that Daniel will be caught, and find how long this will take. (iv) Show that the path taken by the lion is a circle. (v) For the special case in which $U = u$, sketch the path taken by the lion and find the point of capture.

6. A bee flies on a trajectory such that its polar coordinates at time t are given by
$r = \frac{bt}{\tau^2}(2\tau - t)$ and $\theta = \frac{t}{\tau}$; $(0 \leq t \leq 2\tau)$
where b and τ are positive constants. Find the velocity vector of the bee at time t. Show that the least speed achieved by the bee is b/τ. Find the acceleration of the bee at this instant.

*Note: Please follow the strategies for "Problem Solving" explained in the class.