

Indian Institute of Technology Patna
Department of Electrical Engineering
EE381 - Power Systems

Autumn - 2024

End Semester Exam - Solution.

November 29, 2024

There are 5 questions. They carry equal marks.

(5 × 10 = 50)

1. For a simple four bus system, the line admittances are given below:

Y_{bus}	Bus Code	Admittance
$3-j12$	1-2	$2-j8$
$-2+j8$	1-3	$1-j4$
$-1+j4$	2-3	$0.666-j2.664$
0	2-4	$1-j4$
	3-4	$2-j8$

The schedule of active and reactive power is given below:

	Bus	P	Q	V	Type
$P_2 = -0.5$	1	-	-	1.06	Slack
$Q_2 = -0.2$	2	0.5	0.2	-	PQ
$P_3 = -0.4$	3	0.4	0.3	-	PQ
$Q_3 = -0.3$	4	0.3	0.1	-	PQ
$P_4 = -0.3$					
$Q_4 = -0.1$					

Assuming a flat voltage start, determine the voltage at the end of first iteration using Gauss-Seidal method. Take $\alpha = 1.6$. (α is acceleration factor.)

$$V_2' = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0 \right]$$

$$V_2' = 1.01187 - j0.028$$

$$V_2'_{acc} = V_2^0 + \alpha (V_2' - V_2^0)$$

$$= 1.01899 - j0.046$$

$$V_3' = 0.9741 - j0.029 \quad \therefore V_3' = \frac{1}{Y_{33}} \left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1 - Y_{32}V_2' - Y_{34}V_4^0 \right]$$

$$V_3'_{acc} = 0.97051 - j0.04479$$

$$V_4' = \frac{1}{Y_{44}} \left[\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1 - Y_{42}V_2' - Y_{43}V_3' \right]$$

$$V_4' = 0.9716 - j0.064 \text{ (1)}$$

$$V_4'_{acc} = 0.9545 - j0.1034$$

2. The fuel inputs per hour of plants 1 and 2 are as follows:

$$F_1 = 0.2P_1^2 + 40P_1 + 120 \text{ Rs/hr}$$

$$F_2 = 0.25P_2^2 + 30P_2 + 150 \text{ Rs/hr}$$

If the maximum and minimum loading on each unit is 100 MW and 25 MW, the demand is 180 MW and the transmission losses are neglected, determine

- the economic operating schedule and the corresponding cost of generation.
- the savings obtained by loading the units as per equal incremental production cost, if the load is equally shared by both the units.

a) For economic operation,

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$$

$$\begin{aligned} \frac{dF_1}{dP_1} &= 0.4P_1 + 40 \quad \text{--- (1)} \\ \frac{dF_2}{dP_2} &= 0.5P_2 + 30 \quad \text{--- (2)} \end{aligned} \quad \text{--- (1) = (2)}$$

$$\left. \begin{aligned} 0.4P_1 + 40 &= 0.5P_2 + 30 \\ P_1 + P_2 &= 180 \text{ MW} \end{aligned} \right\} \text{ on solving them,}$$

$$P_1 = 88.89 \text{ MW}, \quad P_2 = 91.11 \text{ MW}$$

The cost of generation

$$\begin{aligned} F_T &= F_1 + F_2 = 5285.88 + 4958.55 \\ F_T &= 10244.43 \text{ Rs/hr} \end{aligned}$$

b)

As per equal loading
 $P_1 = 90 \text{ MW}, \quad P_2 = 90 \text{ MW}$

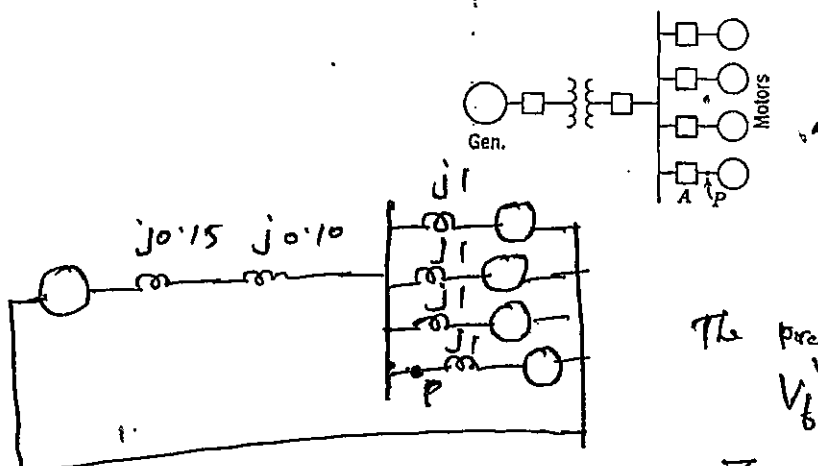
$$\begin{aligned} F_T &= F_1 + F_2 \\ &= 5340 + 4875 = 10215 \text{ Rs/hr} \end{aligned}$$

$$\begin{aligned} \therefore \text{Savings} &= 10215 - 10244.43 \\ &= 0.57 \text{ Rs/hr} \end{aligned}$$

3. A 25 MVA, 13.8 kV generator with $X_d'' = 15\%$ is connected through a 25 MVA, 13.8/6.9 kV transformer with leakage reactance of 10% to a bus which supplies four identical motors as shown below. The sub-transient reactance of each motor is 20% on a base of 5 MVA and 6.9 kV. The bus voltage at the motors is 6.9 kV when a three phase fault occurs at point P. Find

in Amperes

- (a) the sub-transient current in the fault.
(b) the sub-transient current in the breaker A.



Base MVA = 25 MVA
Base kV for the Generator
= 13.8 kV

Base kV for the motor side
= 6.9 kV

∴ The motor reactances

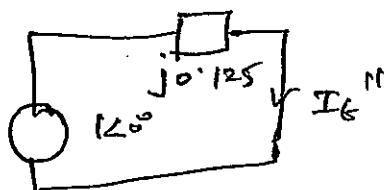
are
 $X_d'' = 0.2 \times \frac{25}{5} = 1 \text{ pu}$

The prefault voltage
 $V_f = 1.0 \angle 0^\circ \text{ pu}$

$Z_{Th} = \frac{j0.25 \times j0.25}{j0.5}$

$Z_{Th} = j0.125 \text{ pu}$

a)



$I_f'' = \frac{1 \angle 0^\circ}{j0.125} = -j8 \text{ pu}$

The bar current in the 6.9 kV circuit is (Motor)

$|I_b| = \frac{25 \times 10^3}{\sqrt{3} \times 6.9} = 2090 \text{ A}$

∴ $|I_b| = 8 \times 2090 = 16,720 \text{ A}$

- b) The Generator contribution is ~~50%~~ (50% of the fault) current

$= -j8 \times \frac{j0.25}{j0.5} = -j4 \text{ pu}$

The three motors' contribution is

$= -j4 \times \frac{j0.25}{j0.375} = -j3 \text{ pu}$

∴ The subtransient current in the Breaker A

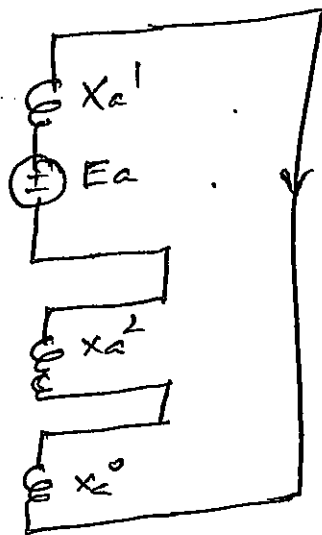
③

$= -j7 \text{ pu}$

$I'' = 7 \times 2090 = 14,630 \text{ A}$

4. The positive, negative and zero sequence reactances of a 25 MVA, 13.2 kV alternator are 0.3 p.u, 0.2 p.u and 0.1 p.u, respectively. The generator is star connected and neutral is solidly grounded. When it is unloaded, find the fault current in Ampere and line-line voltages in kV when a fault of single line to ground fault occurs at the terminals.

SLG Fault



$$E_a = 1 \text{ p.u.}$$

$$I_a^1 = I_a^2 = I_a^0 = \frac{I_f}{3}$$

$$I_f = 3 \times \frac{E_a}{X_{a1} + X_{a2} + X_{a0}} = \frac{1 \angle 0^\circ \times 3}{j0.6}$$

$$I_f = 5 \angle -90^\circ \text{ p.u.}$$

$$I_a^1 = \frac{5}{3} \angle -90^\circ \text{ p.u.}$$

I_f in A

$$= 5 \times \frac{25 \times 10^3}{\sqrt{3} \times 13.2}$$

$$I_f = 5.4673 \text{ KA}$$

$$V_{a1} = E_a - I_a^1 \times jX_{a1}$$

$$= 1 \angle 0^\circ - \frac{5}{3} \angle -90^\circ \times j0.3$$

$$V_{a1} = 0.5 \text{ p.u.}$$

$$V_{a2} = -I_a^2 \times jX_{a2}$$

$$= -\frac{5}{3} \angle -90^\circ \times j0.2$$

$$V_{a2} = -\frac{1}{3} \text{ p.u.} = -0.3333 \text{ p.u.}$$

$$V_{a0} = -I_a^0 \times jX_{a0} = -\frac{5}{3} \angle -90^\circ \times j0.1$$

$$V_{a0} = -0.1667 \text{ p.u.}$$

$$V_a = V_{a1} + V_{a2} + V_{a0} = 0$$

$$V_b = V_{b1} + V_{b2} + V_{b0}$$

$$= a^2 V_{a1} + a V_{a2} + V_{a0}$$

$$V_b = -0.25 - j0.72$$

$$V_c = a V_{a1} + a^2 V_{a2} + V_{a0}$$

$$\textcircled{4} = -0.25 + j0.72$$

$$V_{ab} = 10.08 \text{ KV}$$

$$V_{bc} = 19.05 \text{ KV}$$

$$V_{ca} = 10.08 \text{ KV}$$

$$V_{a1} = 0.25$$

$$+j0.72$$

$$V_{b1} = -j1.44$$

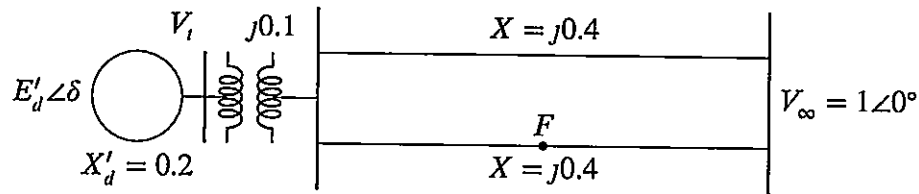
$$V_{c1} = -0.25 + j0.72$$

$$V_{a2} = 0 - V_b = -V_b$$

$$V_{b2} = V_b - V_c$$

$$V_{c2} = V_c$$

5. A 50 Hz synchronous generator having inertia constant $H = 5$ sec and a direct axis reactance $X'_d = 0.2$ p.u. is connected to an infinite bus through a transformer and a double circuit line. The machine is delivering 1 per unit power with a terminal voltage $|V_t| = 1.0$ per unit.



A three phase fault occurs at F . The fault is cleared by opening the faulted line. Determine the critical clearing angle.

$$P = 1 \text{ pu} \quad P = \frac{V_t V_\infty}{0.3} \sin(\angle V_t - \angle V_\infty)$$

$$\therefore \angle V_t - \angle V_\infty = \sin^{-1}(0.3) = 17.458^\circ \quad (0.3047 \text{ rad})$$

$$\bar{I} = \frac{1 \angle 17.458^\circ - 1 \angle 0^\circ}{j0.3} = 1 + j0.1525 = 1.012 \angle 8.73^\circ$$

$$\bar{I} = 1.012 \angle 0.152 \text{ rad.}$$

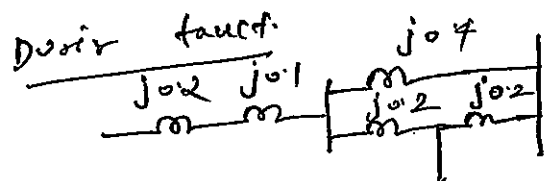
$$E'_d \angle \delta = V_t \angle 17.458^\circ + \bar{I} \times j0.2$$

$$= 0.9232 + j0.5 = 1.05 \angle 28.44^\circ = 1.05 \angle 0.496 \text{ rad}$$

Pre-fault

$$X = j0.5$$

$$P_{\text{max}} = \frac{1.05 \times 1}{0.5} = 2.10$$



After Y-Δ-Y conversion,

$$X_{II} = j1.3$$

$$P_{\text{max II}} = 0.8$$

Post fault

$$\delta_0 = 0.465 \text{ rad}$$

$$\delta_{\text{max}} = \pi - \sin^{-1}\left(\frac{P_m}{P_{\text{max III}}}\right)$$

$$\delta_{\text{max}} = 2.412 \text{ rad}$$

$$X_{III} = j0.7$$

$$P_{\text{max III}} = 1.5$$

$$P_m = P_e = 1 \text{ pu}$$

$$\delta_{cr} = \cos^{-1} \left[\frac{P_m (\delta_{\text{max}} - \delta_0) + P_{\text{max III}} \cos \delta_{\text{max}} - P_{\text{max II}} \cos \delta_0}{P_{\text{max III}} - P_{\text{max II}}} \right] \quad (5)$$

$$\delta_{cr} = 1.4435 \text{ rad} / 82.726^\circ$$